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# LES-4
# Spin Axis Orientation System

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

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## ABSTRACT

This report deals with the design of a satellite stabilization system to maintain the angular momentum of a spin stabilized satellite normal to the satellite's orbital plane. This orientation of the angular momentum allows a higher gain (narrower beam) antenna in the X-band communications link in the satellite. The stabilization system uses the Earth's magnetic field as a source of torque and requires no commands from the ground as all the error sensing components are satellite-borne.

Accepted for the Air Force
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Chief, Lincoln Laboratory Office

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# LES-4 SPIN AXIS ORIENTATION SYSTEM

## 1. Introduction

A stabilization system proposed by B. Howland is to be tested on LES-4. Its purpose is to point the satellite angular momentum, $\vec{J}$, along the direction of the unit orbit normal, $\vec{N}$, using the interaction of the satellite magnetic moment, $\vec{M}$, with the earth's magnetic field, $\vec{B}$, to generate the torque, $\vec{\tau}$, necessary to produce the required changes in angular momentum. LES-4 is to be flown in a quasi synchronous equatorial orbit of 18,200 n. mi. altitude.

## 2. Dynamic Considerations

The details of the motion of a complex rigid body about its center of mass are being considered by B. Moriarty, so this report will deal with the sensors and electronics necessary to produce and control the satellite magnetic moment. The analysis of the motion is based on the "Fast Top" approximation which assumes that the principle moments of inertia A, B, C, of the body obey the relationships:

$$A = B, \qquad C/A > 1$$

Under these conditions nutation is small, the angular momentum, $\vec{J}$, and the angular velocity, $\vec{\omega}$, of the body are collinear, and the body finds itself in a stable rotation when the axis of the large moment of inertia, C, is also directed along $\vec{J}$ and $\vec{\omega}$. Then the angular momentum and angular velocity are simply related.

$$\vec{J} = C\,\vec{\omega}$$

If such a satellite is in an earth orbit and we wish to force its angular momentum, $\vec{J}$, to lie along $\vec{N}$, we may do so by removing that component of $\vec{J}$ perpendicular to $\vec{N}$, $\vec{J}_{\perp}$, and leave the satellite with a somewhat reduced angular momentum, $\vec{J}_{11}$, which is in the required direction.

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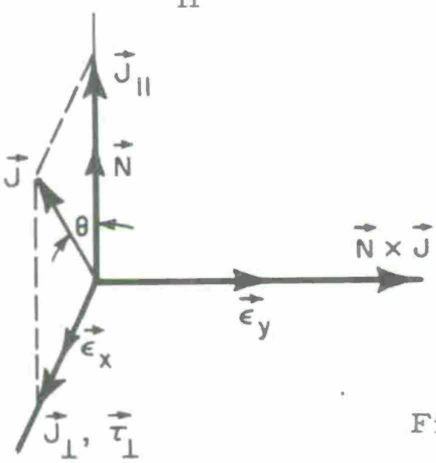


$$\vec{J} = \vec{J}_{11} + \vec{J}_{\perp}$$

$$\vec{J}_{11} = (\vec{J} \cdot \vec{N})\,\vec{N}$$

$$\vec{J}_{\perp} = (\vec{N} \times \vec{J}) \times \vec{N} = J \sin\theta\,\vec{\epsilon}_x$$

$$= J_{11} \operatorname{Tan}\theta\,\vec{\epsilon}_x$$

Figure 1.

These components are defined in Fig. 1. If the angle between $\vec{N}$ and $\vec{J}$ is $\theta$

$$|\vec{J}_{11}| = \vec{J} \cdot \vec{N} = J \cos\theta$$

$$|J_{\perp}| = |(\vec{N} \times \vec{J}) \times \vec{N}| = J \sin\theta = J_{11} \operatorname{Tan}\theta$$

To remove $\vec{J}_{\perp}$ we need to create a torque $\vec{\tau}_{\perp}$ along $\vec{\epsilon}_x$. We find that the rate of change of $\theta$ is dependent on the strength of $\vec{\tau}_{\perp}$, for if

$$\vec{\tau}_{\perp} = |\vec{\tau}_{\perp}| \vec{\epsilon}_1 = \frac{d}{dt} \quad \vec{J}_{\perp} = \vec{\epsilon}_1 \frac{d}{dt} (J_{11} \tan \theta)$$

$$= \vec{\epsilon}_1 \left(\tan \theta \frac{dJ_{11}}{dt} + J_{11} \frac{d}{dt} \tan \theta\right)$$

and

$$\vec{\tau}_{11} = 0 = \vec{N} \frac{d}{dt} (J_{11})$$

so

$$\frac{d}{dt} J_{11} = 0$$

$$|\vec{\tau}_{\perp}| = \tan \theta \frac{dJ_{11}}{dt} + J_{11} \frac{d}{dt} \tan \theta$$

$$|\vec{\tau}_{\perp}| = \frac{J_{11}}{\cos^2 \theta} \frac{d\theta}{dt}$$

or

$$\frac{d\theta}{dt} = \frac{|\vec{\tau}_{\perp}| \cos^2 \theta}{J_{11}}$$

If $\theta$ is small $\cos \theta \simeq 1$ and

$$\frac{d\theta}{dt} = \frac{|\vec{\tau}_{\perp}|}{J_{11}}$$

Reducing $\theta$ to zero by the action of $\vec{\tau}_{\perp}$ leaves the satellite with $|\vec{J}_{11}| = J \cos \theta$ units of angular momentum directed along $\vec{N}$ as desired.

## 3. System Configuration (General)

### 3.1 Error Sensing

Consider the following system for sensing the component of angular momentum perpendicular to the orbit normal and reducing that component in magnitude:

The satellite carries four sun sensors that produce an output only when the radiative energy from the sun illuminates a narrow slit parallel to the satellite angular momentum, $\vec{J}$. These sensors S1 → S4 are placed symmetrically around the satellite and a magnetic-moment producing electromagnet (labelled $\alpha \rightarrow \delta$) is associated with each sensor.

The satellite carries one composite earth sensor, E, that produces an output only when radiative energy from the Earth illuminates a slit parallel to the satellite angular momentum, $\vec{J}$, placed between S4 and S1 on the satellite skin and the sensor produces different signals according to whether the earth appears to be above (H), below (L), or in (A) the plane perpendicular to the satellite angular momentum. See Figs. 2 and 3.

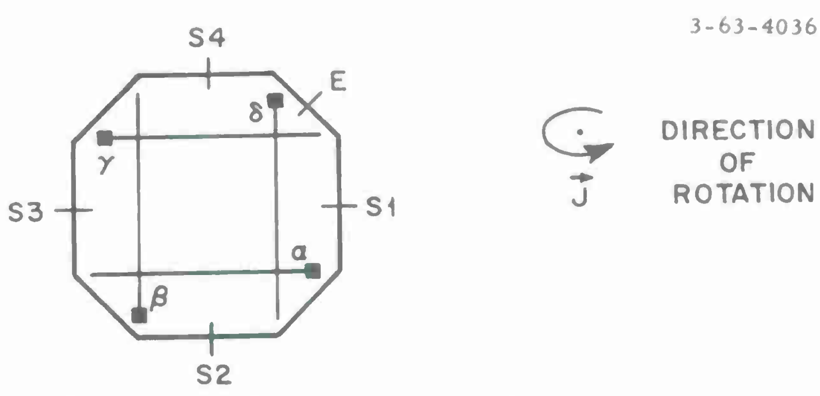


Figure 2. Layout of Earth and Sun sensors and electromagnets in LES-4 as viewed from top of satellite. The $\alpha$ end of the $\alpha$ rod shall be defined as + when the magnetic moment, $\vec{M}_{\alpha}$, is directed toward that end of the rod.

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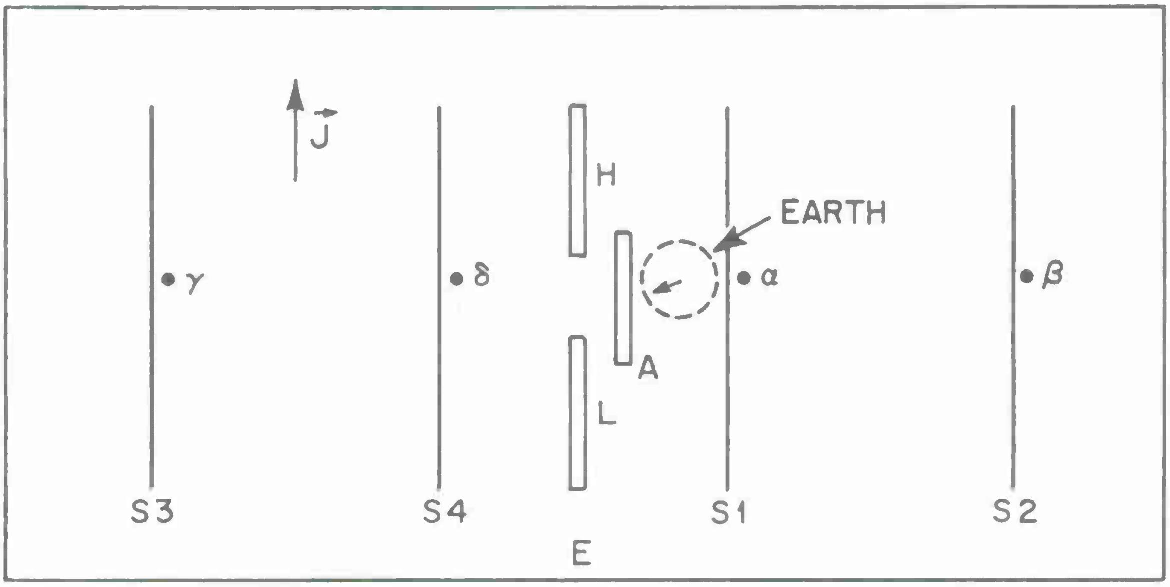


Figure 3. Layout of Earth and Sun sensors and electromagnets in LES-4 as viewed by Mercator's projection technique from inside. Arrow denotes motion of Earth as seen "strobed" by Sun signals.

Once per satellite rotation the Sun sensors are activated in the order S1, S2, S3, S4, S1...etc. Once per satellite orbit the sun sensor S2 is activated at the same instant that the earth activates the earth sensor E. This coincidence defines the x axis in inertial space as shown in Fig. 4. The coincidence S3 and E defines a y axis similarly. These coincidences occur approximately $90^{\circ}$ apart from one another in the orbit, hence, the x and y axis are approximately orthogonal to each other and to $\vec{N}$, the orbit normal. Then the unit vectors $\vec{\epsilon}_x$, $\vec{\epsilon}_y$, $\vec{N}$ define a right-handed inertial coordinate system with its origin at the center of the earth.

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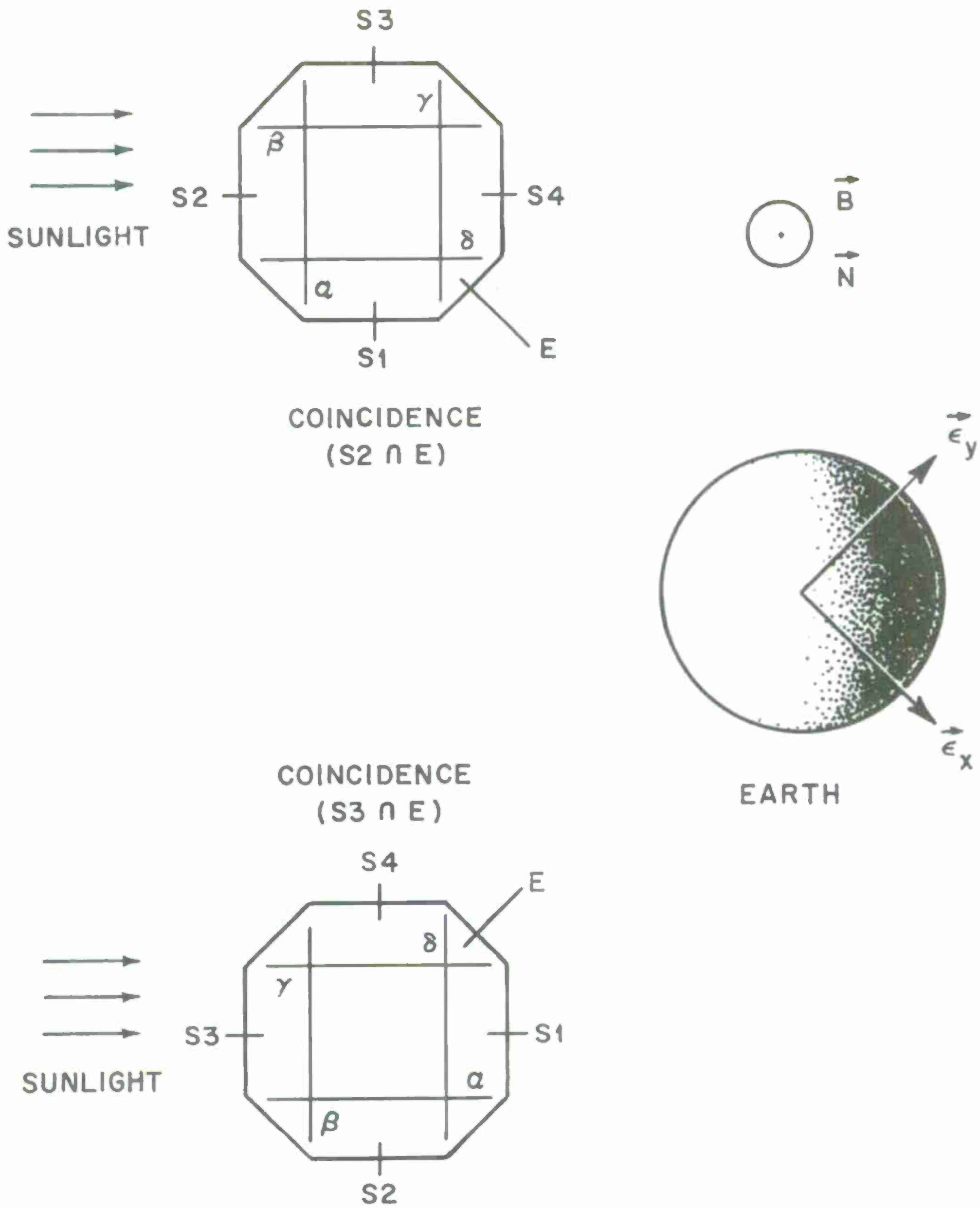


Definition of x, y, N inertial coordinate system by $(S2 \cap E)$ and $(S3 \cap E)$ coincidences as viewed from $\vec{N}$.

Figure 4.

The existence of components of $\vec{J}$ along the x and y directions is determined as follows: When the coincidence ($S2 \cap E$) occurs, if H is illuminated there must be a positive component of $\vec{J}$ along the x direction. Similiarly, if A is illuminated $J_x = \vec{J} \cdot \vec{\epsilon}_x = 0$ or if L is illuminated $J_x$ is negative. Mixtures of H and A or A and L signals will be considered later.

Coincidence of S3 and these signals measure the polarity of $J_y = \vec{J} \cdot \vec{\epsilon}_y$.

The satellite is oriented properly (with $\vec{J}$ and $\vec{N}$ collinear) when both $J_x$ and $J_y$ are zero. Note that $\vec{\epsilon}_x$ and $\vec{\epsilon}_y$ need not be strictly orthogonal.

### 3.2 Error Correction

Returning to Fig. 4 and noting that the Earth's magnetic field, $\vec{B}$, always has a significant component, $B_N$, in the $\vec{N}$ direction we may generate a torque $\vec{\tau}_x$ along the x direction with a magnetic moment Ml along the y direction since

$$\vec{\tau} = \vec{M} \times \vec{B}$$

substituting

$$\vec{M} = Ml\ \vec{\epsilon}_y,\ \vec{B} = B_N\ (\vec{N})$$

we find

$$\vec{\tau} = Ml\ B_N\ [\vec{\epsilon}_y \times (\vec{N})] = Ml\ B_N\ \vec{\epsilon}_x$$

This torque tends to increase $J_x$. If Ml is negative along the y axis it produces a negative torque along the x axis which tends to reduce $J_x$.

The satellite can be oriented ($J_x = 0$) if the information concerning the sign of $J_x$ is used to turn on M1 with the appropriate sign,

| $J_x$ | M1 correction necessary |
|---|---|
| O | O |
| + | - |
| - | + |

We note that M1 is a magnetic moment along a fixed inertial direction (y) which must be generated by an electromagnet attached to a rotating satellite. If we consider only the $\alpha$-electromagnet we notice that it will produce a positive magnetic moment along M1 if it is positive from the time sunlight strikes S3 until it strikes S4 and again if it is negative during the S1 to S2 interval. During the S2 and S3 and S4 to S1 intervals, the $\alpha$ electromagnet is more closely aligned with the x inertial direction and can be used to create a correction for $J_y$ rather than $J_x$. Similar reasoning may be employed to explain the operation of the $\beta$ - $\delta$ rods.

Hence, commutation of the electromagnets by the signals from the sun sensors can produce correction torques along the x and y axes simultaneously.

## 3.3 Circuit Description and Design

### 3.3.1 Command Generation

The coincidences that define the x and y directions and measure the angular momenta $J_x$ and $J_y$ occur only once an orbit, but the information must be used to generate the commutation signals for the entire orbit. This can be accomplished by setting 4 latching relay flip-flops (labeled A-D) from the coincidence signals. The information about $J_x$ derived from the $(S2 \cap E)$ coincidence is stored in the A and B flip-flop relays as shown in Table II below.

| S2 coincident with E and | | | $J_x$ | Set FFA To | Set FFB To | MI correction necessary |
|---|---|---|---|---|---|---|
| $\bar{H}$ | $\bar{A}$ | $\bar{L}$ | no info | - | - | ? |
| $\bar{H}$ | $\bar{A}$ | L | + | 0 | 1 | - |
| $\bar{H}$ | A | $\bar{L}$ | 0 | 1 | 1 | 0 |
| $\bar{H}$ | A | L | + | 0 | 1 | - |
| H | $\bar{A}$ | $\bar{L}$ | - | 1 | 0 | + |
| H | $\bar{A}$ | L | 0 | 1 | 1 | 0 |
| H | A | $\bar{L}$ | - | 1 | 0 | + |
| H | A | L | 0 | 1 | 1 | 0 |

TABLE II

State Table for Signals to Set Relay Flip-Flops A and B to Store $J_x$ Information

Examination of Table II shows that the occurrence of any of 5 separate coincidences requires flip-flop A to be set to 1. The coincidences are

Set FFA to 1 if: $S2 \cap [HAL \cup HA\bar{L} \cup H\bar{A}L \cup H\bar{A}\bar{L} \cup \bar{H}A\bar{L}]$

A little Boolean algebra reduces the required signal to

Set FFA to 1 if: $S2 \cap [H \cup A\bar{L}]$

similarly

Set FFA to 0 if: $S2 \cap \lceil \bar{H}L \rceil$

and

Set FFB to 1 if: $S2 \cap [L \cup A\bar{H}]$

Set FFB to 0 if: $S2 \cap \lceil H\bar{L} \rceil$

The C and D flip-flops store the $J_y$ information and require the coincidence of S3 with the composite signals in brackets above. An expeditious method of setting the A-D flip-flops is to generate the composite signals from the H, A, and L signals and then form the S2 and S3 coincidences to set the flip-flops to new states. [The latching relay flip-flops (A-D) require a large power surge to operate, but circuitry is provided to energize them only if they are in the "wrong" state at the time of a command. This feature saves multiple triggering of the relays and helps to insure that the relays will operate reliably on command.]

A block diagram of the circuitry required to store the $J_y$ information in the A and B relay flip-flops is shown in Fig. 5

### 3.3.2 Switching Sequence Generator

The switching sequences for the electromagnets may be generated by a shift register into which is loaded the information from the relay flip-flops A-D. If we consider only the $\alpha$ electromagnet we notice in Table II and Fig. 4 that any magnetic moment that $\alpha$ creates during the S1-S2 interval is the same polarity as FFA, and during S3-S4 interval is the polarity of FFB. Consequently during S2-S3 the $\beta$ electromagnet is to be the same polarity as FFA, druing S3-S4 the $\gamma$ electromagnet is the same as FFA, etc.

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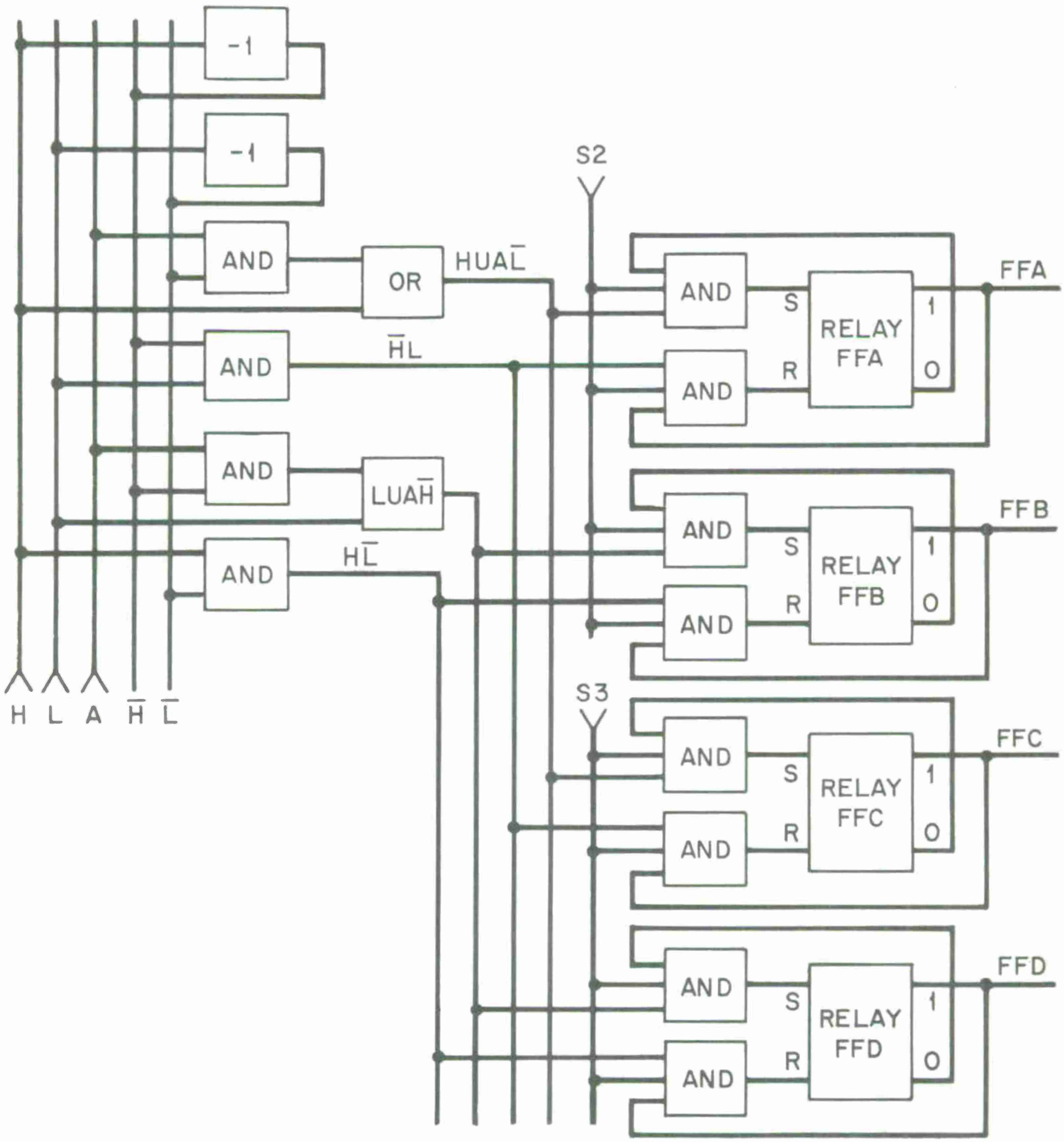
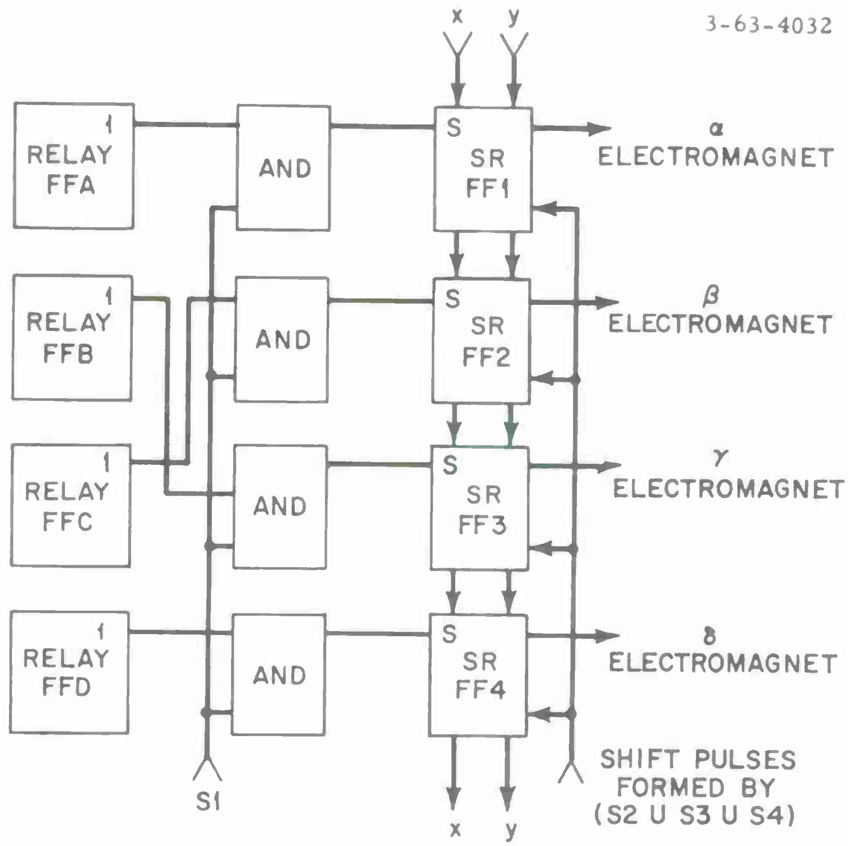



Figure 5. Block diagram of storage logic.

Consider then, the following circuit.




On the S1 pulse the contents of the A-D flip-flops is loaded into the four shift registers [1→4] and used to set the polarities of the four electromagnets. When S2 occurs the satellite has rotated $90^{\circ}$ in inertial space and the shift register shifts the information one register "back" so that the β electromagnet is now set to the polarity of the α electromagnet and so on as the satellite rotates. This circuit insures that the electromagnet most closely aligned with an inertial axis is programmed to have the polarity required for the orientation desired.

Since the $\alpha$ electromagnet with fixed magnetic moment $\pm M_{\alpha}$, is not perfectly aligned with the inertial directions continuously, there is an averaging effect along the inertial directions. For instance, during the S1-S2 interval if $M_{\alpha}$ is positive we find the average of the moments in inertial space to be

$$<M1> = \frac{2\sqrt{2}}{\pi} M_{\alpha} + \text{terms from } M_{\beta} \rightarrow M_{\delta}$$

and

$$<M2> = 0 M_{\alpha} + \text{terms from } M_{\beta} \rightarrow M_{\delta}$$

[The averages are valid only in the "Fast Top" approximation with $|\vec{\tau}| = |MB| << |J|$, for the actual motion may have a precession or nutation of the body forced by the non-linear coupling of momenta and velocities].

4. System Configuration (Specific)

The satellite to be oriented has a maximum moment of inertia, C, of about 2.5 kg-m$^2$ and will be injected into a circular quasi synchronous equatorial orbit spinning at 12 rpm and will therefore have an angular momentum of about $\pi$ kg-m$^2$/sec. The earth's magnetic field near synchronous altitude varies between 100 and 200 gamma or 1 to 2 x 10$^{-7}$ Webers/m$^2$.

Choosing the erection rate $(\frac{d\theta}{dt})$ to be 3°/day [6.09 x 10$^{-7}$ ra/sec] along either the x or y axes fixes the magnitude of the average magnetic moment, <M1>, for if

$$< \frac{d\theta}{dt} > = < \frac{|\tau_{\perp}|}{|J|} > \cong \frac{<Ml> <B>}{|J|}$$

then

$$< Ml > \cong \frac{|J| < \frac{d\theta}{dt} >}{<B>}$$

and using $< B > = 10^{-7}$ $w/m^2$ to allow a safety factor

$$< Ml > \cong \frac{\pi \; 6.09 \cdot 10^{-7}}{10^{-7}}$$

$$\cong 6.09 \, \pi \text{ ampere - meter}^2$$

The magnetic moment <Ml> is always generated by the simultaneous commutation of 2 electromagnets [i.e. $M_\alpha + M_\gamma$] each of which must generate

$$M_\alpha = M_\gamma = \frac{< Ml >}{2} \cdot \frac{\pi}{2\sqrt{2}}$$

$$= \frac{6.09 \, \pi^2}{4\sqrt{2}} = 10.6 \text{ a-m}^2$$

of magnetic moment.

In a separate report, W. L. Black deals with the optimization of the supermendur core electromagnets used to produce the satellite borne magnetic moment and a separate memo will use these general results to show the design procedure subject to the system constraints [i.e., wire size, weight, voltages available] and describe the final circuit configuration.

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